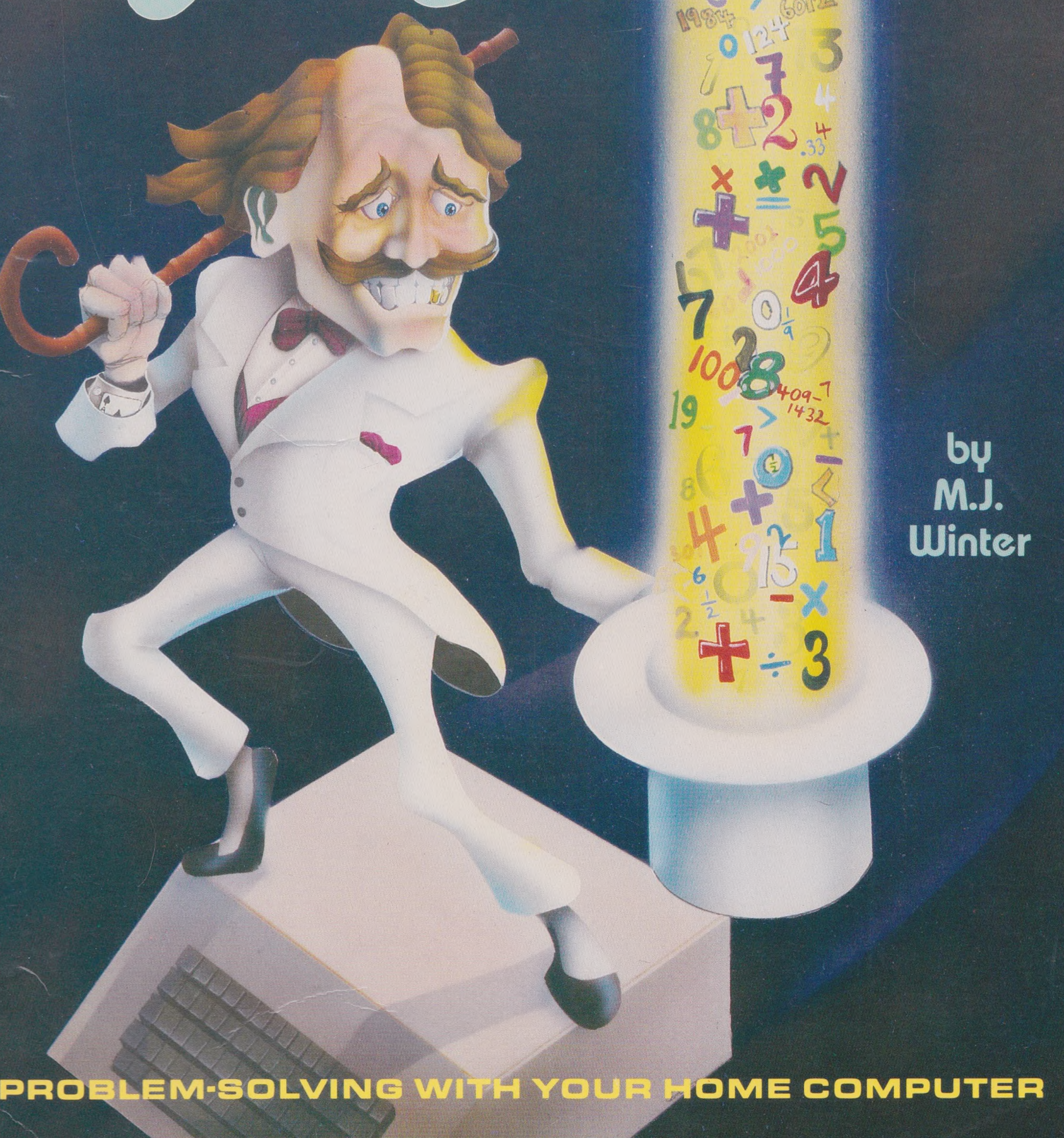
Compumath Magic

$7.95

by M.J. Winter



PROBLEM-SOLVING WITH YOUR HOME COMPUTER

Compumath

Magic

Compumath Magic

M.J. Winter, Ph.D.
Professor of Mathematics
Michigan State University

Illustrated by
Martin Cannon

DATAMOST™ INC
8943 Fullbright Ave., Chatsworth, CA 91311-2750
(818) 709-1202

DATAMOST™ INC

ISBN 0-88190-217-9

TABLE OF CONTENTS

Wrap-Ups:

PREFACE

There's a difference between knowing BASIC and knowing how to use it to solve problems. *COMPUMATH MAGIC* is intended for "intermediate programmers" who want to learn how to use BASIC applications.

The intermediate programmer should know the meaning of most of these commands: PRINT, GOTO, IF . . . THEN, FOR . . . NEXT, INPUT, INT, SQR, RND, MID$, and LEN. Other commands, such as ABS, DIM, VAL, and STR$, are reviewed in the text as they are needed. A working knowledge of 6th grade mathematics is needed for most of the problems.

The book consists of lessons, each containing WARM-UP exercises, a Problems set and later, a corresponding WRAP-UP section. The problems in each set are related, but progress in complexity. Each set begins with a WARM-UP exercise, which will be very helpful to those with minimal computer experience or a minimal mathematical background. In most sets, the first problem is a direct application of the Warm-Up. Advanced users may not need the hints and suggestions, but even they should heed the occasional warnings.

Succeeding problems require moderate, then greater modifications of the original program. Most require thought and planning. The earlier problems will have demonstrated *how* to carry out certain procedures. The later problems require decisions about what procedures are needed.

The setting and level of the problems make them interesting and accessible to intermediate programmers. Some will use the computer equivalent of brute force on the harder problems, others will delight in making the computer do as much work as possible, as efficiently as possible. The mathematical and computer concepts presented in this book are basic. They provide a framework which can be expanded to other applications.

Following the Problems is the Wrap-Up chapter. The Wrap-Up for a problem set provides historical background, explanations, and/or extensions of the problems. The Wrap-Up includes additional problems which are usually more challenging.

For those needing help getting started, or wanting confirmation of their results, a disk with at least one program for each problem set is available. See page 93.

Books of computer problems are often hodgepodge collections. The grouping of the problems in this book should make some users more aware of the structure of mathematics, the power and applicability of mathematical techniques, and of course, the uses of a computer in solving problems.

The material in this book has been used with widely varied groups ranging from 11-year-olds, to experienced high school math teachers. Although their mathematical and computing backgrounds differed greatly, all were successful in solving most of the problems in each set.

PROBLEM SOLVING WITH COMPUTERS

Three ways computers are used in mathematics and science are to exhaust a large number of possibilities, to perform lengthy and difficult computations, and to generate data which can be used as the basis for theories. All of these uses are represented in this book.

Exhaustive Search: By considering all possible combinations of nickels, dimes, and pennies, one can find those combinations that total exactly 25 cents. However, to use the search method, it is necessary to know the limits of where the solution may lie. To make change for 25 cents, no more than 25 pennies, 5 nickels, and 2 dimes can possibly be needed. Finding appropriate limits on FOR . . . NEXT loops is a challenge. The more possible solutions to be checked, the longer the program will take to run. The time needed to find all ways to make change for a quarter is considerably shortened if the programmer realizes that the number of pennies must be a multiple of five.

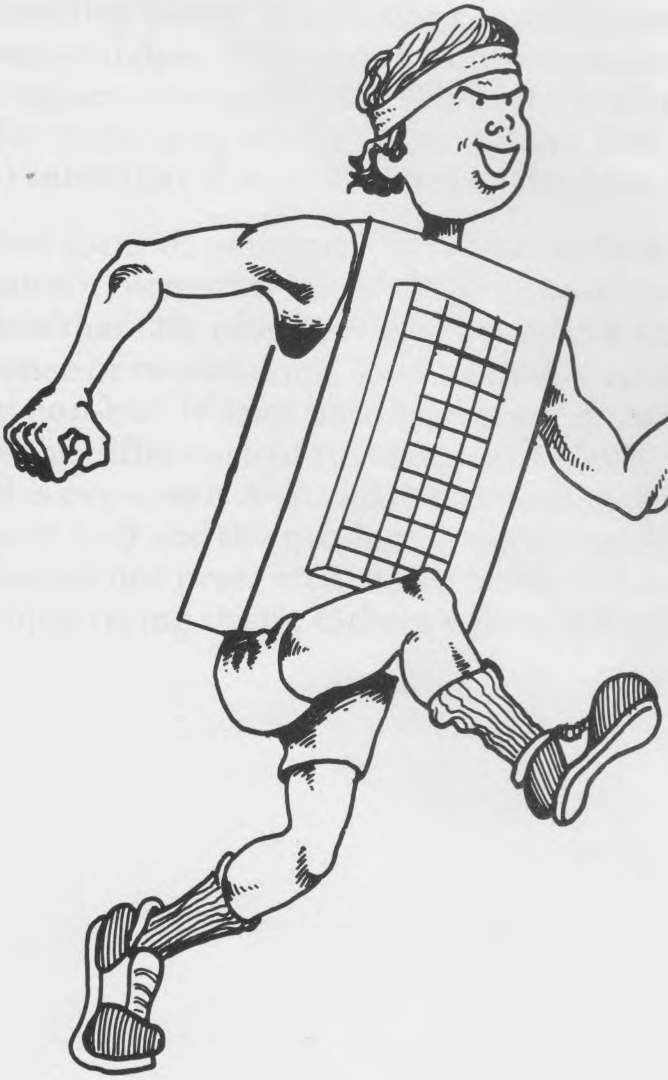
Mathematicians may point out that there are classical techniques for solving many of these problems. Those with one or two years of college mathematics may have learned how to use congruences to solve Diophantine equations, the Euclidean algorithm to find greatest common divisors, and matrices to find the solution to systems of linear equations. However, these tools are not available to everyone. (They weren't available to those who first solved some of the problems either.) Nor are the advanced techniques necessarily faster on the computer. Exhaustive search enables even middle school students to solve these problems.

Lengthy Computations: Some problems involve so many divisions or other arithmetic operations, that few people can solve them without making errors. If the errors confuse or obscure the solutions, nothing has been gained by the tedious work. Experience shows that even the "Locker Door" problem is subject to careless mistakes. The problems in the set called "Diminishing Returns" can be done using a calculator, but not by hand; too many significant digits are required for an accurate final answer.

Generating Data: Simulations, such as coin tossing, are one form of generated data. Numerous simulations can lead to good approximate values of probabilities. If hundreds of rolls of two dice all show doubles occurring about 1/6th of the time, then we may (fairly safely) infer that the probability of doubles is about 1/6.

Another form of generated data occurs in looking at the numbers that cannot be written as the difference of two squares: of the numbers less than 20, only 2, 6, 10, 14, and 18 cannot be written as the difference of two squares. These numbers can all be written as twice an odd number. It's not hard to prove that twice an odd number cannot be the difference of two squares. (Write $A^2 - B^2$ as [A+B]*[A–B]. If A+B is even, so is A–B and their product is divisible by 4. If A+B is odd, so is A–B and the product is not divisible by 2.) While "proofs" like this are not presented in this book, the amateur mathematician may enjoy trying them. Others will stop with classifying their data.

SHOPPING PROBLEMS



SHOPPING PROBLEMS

WARM-UP Suppose pencils cost 32 cents and notebooks cost 88 cents. If you buy 5 pencils and 3 notebooks, the cost is:

C = 32*5 + 88*3 cents.

If you buy P pencils and N notebooks, the cost is:

C = 32*P + 88*N cents.

If you spent $11.04 on pencils and notebooks, you couldn't have bought more than 1104/32 pencils; you might not have bought any. You couldn't have bought more than 1104/88 notebooks.

Enter and RUN this program. Copy the output.

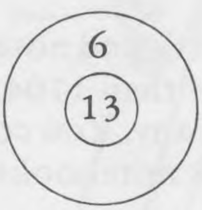
```
10 PRINT "PENCILS AND NOTEBOOKS"
20 FOR P = 0 TO 1104/32
30 FOR N = 0 TO 1104/88
40 C = 32*P + 88*N
50 IF C = 1104 THEN PRINT P,N
60 NEXT N
70 NEXT P
```

Problems

1. Pencils are 32 cents, notebooks are 88 cents. You spend $11.12. What possible combinations did you buy?

2. Pencils and notebooks cost the same as in #1. What is the next amount larger than $11.12 that you could have spent exactly?

3. An archer's score on this target was 100:



What possible combinations of arrows landed on the target?

4. Do problem #3 with the total points = 101, 102, 103, 104, and 105.

5. Once six consecutive scores are made, all greater numbers are possible scores. What is the largest total that cannot be made with this target?

REMAINDERS

WARM-UP Look at this problem: $7\overline{)41}$

The Divisor is ________
The Dividend is ________
The Quotient is ________
The Remainder is ________

The equation connecting these is:

41 = 7*____ + _____

If the problem is: $D\overline{)N}$
the quotient is the whole number part of N/D:
Q = INT(N/D)
The connecting equation for D, N, Q, and the remainder R is:
N = D*Q + R
From this equation, the rule for R is:
R = N - D*Q

Use this program to do a division problem:

```
10 PRINT "DIVISOR"
20 INPUT D
30 PRINT "DIVIDEND"
40 INPUT N
50 Q = INT(N/D)
60 R = N - D*Q
70 PRINT "QUOTIENT IS ";Q
80 PRINT "REMAINDER IS ";R
```

Check your program on 1111/7. The remainder is ________.

Problems

1. Find all numbers less than 100 that leave a remainder of 5 when divided by 7.

2. Find all numbers less than 1000 that leave a remainder of 5 when divided by 7 and a remainder of 7 when divided by 8.

3. A robot which had been programmed to count as far as 7, sorted out a keg of bolts. As the bolts were counted, these messages appeared:

   Items counted in groups of 7: 6 leftover
   Items counted in groups of 6: 5 leftover
   Items counted in groups of 5: 4 leftover

   What are the three smallest possible numbers of bolts?

4. Twelve pirates discover a hoard of gold pieces. They agree that at the end of the trip, they will divide the gold among themselves and give any extra pieces to the cook. The cook secretly counts the pieces and observes that there will be exactly one piece leftover. However, if only eleven pirates survive the voyage there will be 6 pieces leftover. If ten pirates share the gold, there will be 9 pieces leftover for the cook. The next day the fish chowder has two pieces of poisoned fish.

   How many pieces of gold were there?

   Vary the numbers of leftover coins — 1,6,9 — to try to find a larger minimum hoard.

ADDING AND AVERAGING

WARM-UP Remember, the computer adds a list of numbers by keeping a running total.

To add the numbers from 1 to 100:

```
1Ø FOR N = 1 TO 1ØØ
2Ø S=S+N
3Ø NEXT N
4Ø PRINT S
```

Problems

1. Add the ODD numbers from 1 to 6, from 1 to 7, from 1 to 9, etc.

   Complete the table, then try to think of a rule.

1+3	=	________
1+3+5	=	________
1+3+5+7	=	________
1+3+5+7+9	=	________
1+3+5+7+9+11	=	________
1+3+5+7 . . . +13	=	________
1+3+5 . . . +15	=	________

   See if you can predict 1+3+5+ . . . +401, then have the computer check it.

2. Write a program that adds all the integers from A to B, where A and B are input variables. RUN the program for:

A	B	Total	
10	21	____	
37	63	____	
____	58	1480	
____	19	135	
3	____	75	
4	____	60	
____	____	90	(sum of 3 terms)
____	____	90	(sum of 5 terms)
____	____	90	(sum of 9 terms)

3. Write an averaging program. The program should ask HOW MANY ITEMS?, then use a FOR . . . NEXT loop of the appropriate size to input and add the numbers. Finally, it should print the average.

RUN the program for: 2,3,0,5,2,4,6,7 ________
and 93,85,82,93,77,66,88 ________

If the last group of numbers was test scores, and there was going to be one more test, what is the lowest grade that would raise the average to 85?

DIMINISHING RETURNS

WARM-UP There are 1000 leaves on the grass. Every time you use the lawn sweeper, you pick up ⅓ of the remaining leaves.

How many passes until there are only 10 leaves left?

Make the computer figure this out. It will print a table listing:
Pass # Leaves Collected Leaves left.

```
10 LV = 1000              starting number is 1000
20 FOR P = 1 TO 12        do 12 passes
30 LC = LV*(1/3)          leaves collected
40 LL = LV – LC           leaves left
50 PRINT P,LC,LL
60 LV = LL                leaves now = leaves left
70 NEXT P
```

To make LC a whole number, add line 35:

```
35 LC = INT(LC+.5)
```

This will round LC to the nearest whole number.

Problems

1. Apple cider is made by repeatedly pressing bags of apples until very little juice is left. Suppose at each pressing, ⅓ of the juice still in the apples is extracted. How many pressings are necessary to extract 80% of the juice? (Let the starting amount of juice = 1. Keep pressing until the juice left is less than .20.) Delete line 35 if you added it.

2. Sue knew ⅓ of her piano piece on Tuesday morning. Each time she practiced she learned ¼ of the remaining part of the piece. How much of the piece did she know after 1 practice? 2 practices? 3? 4? How many practices were necessary for her to know 90% of the piece?

3. Each time Jo's personal stereo is turned on, 1/20 of the remaining power of the batteries is used up. When 95% of the power of the new batteries is gone, there is no sound. If Jo starts out with new batteries, how many times can the stereo be turned on before the batteries are too weak to produce any sound?

4. Kelly's present weight is 74 pounds. Assume Kelly is 12 now, and that each year Kelly's weight will increase by 5% (5%=1/20). How much will Kelly weight at age 19? If she keeps growing at that rate, how much will she weigh at age 45?

5. Your great-aunt put $500 in a savings account for you. Interest varies, but assume the amount in the account increases by 8% each year. What will the balance be after 5 years? 10 years? 15 years?

DIGITS

WARM-UP The digits of the number 87 are 8 and 7. The 8 is in the tens place and the 7 is in the ones place. The value of 87 is 8*10+7. The digit sum of 87 is 8+7=15. The smallest a digit can be is 0; the largest it can be is 9.

Finish this program to print out all the two-digit numbers, in order. (Pretend 01 is a two-digit number.)

```
10 FOR X = Ø TO 9
20 FOR Y = Ø TO 9
50 PRINT X;Y
60 NEXT Y
70 NEXT X
```

Add line 30 to the warm-up program to print V, the value of the number.

Add line 40 to print D, the digit sum of the number.

Problems

1. Are there any two-digit numbers with a value equal to twice the digit sum? Change line 50 to print only these numbers.

2. Find all two-digit numbers with a value = 3*digit sum.

   Find all two-digit numbers with a value = 4*digit sum.

   Find all two-digit numbers with a value = 5*digit sum.

   Find all two-digit numbers with a value = 6*digit sum.

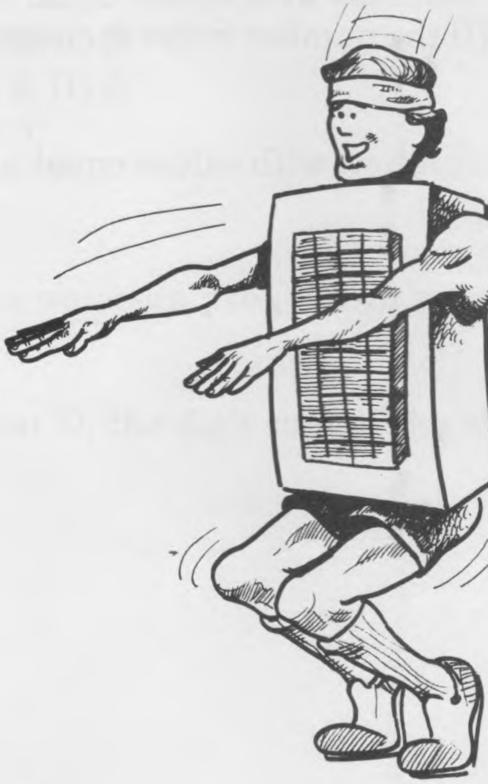
3. Find all three-digit numbers with values equal to the sum of the cubes of the digits. (If the number is xyz then xyz must equal $x^3 + y^3 + z^3$.)

4. Find all three-digit numbers with values equal to the cube of the digit sum.

MAKING CHANGE

WARM-UP If you have D dimes
N nickels
P pennies
their value is

$$V = 10*D + 5*N + P$$



Problems

1. Using dimes, nickels, and pennies, how many different ways can you make 25 cents?

2. Can you make change for $1 using exactly 17 coins (half dollars, quarters, dimes, nickels, pennies)? If so, how?

3. How many different ways are there to make change for a dollar? Print out each way. This program will take a long time to run.

4. It is impossible to make change for a dollar using exactly 99 coins. What other total numbers of coins are impossible?

5. What coins must you carry so you can come up with any amount from $.01 to $1? What is the smallest number of coins?

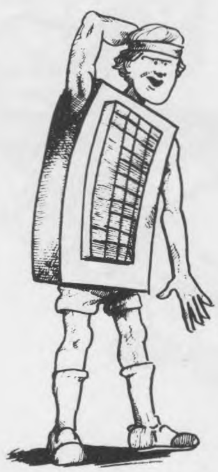
FIBONACCI SEQUENCE

WARM-UP The Fibonacci sequence starts

1, 1, 2, 3, 5, 8, 13, . . .

Each term is the sum of the two that precede it. This program will generate the first 30 terms of the Fibonacci sequence. F(1) is the first term, F(2) is the second term, etc.

```
10 DIM F(30)
20 F(1)=1
30 F(2)=1
40 FOR I = 3 TO 30
50 F(I)=F(I-1) + F(I-2)
60 NEXT I
```



Problems

1. Print a list giving the term number (I) and the value of the term F(I).

   Which terms of the sequence are divisible by 2? (Give their term numbers.)

   Which terms are divisible by 4?
   Which terms are divisible by 5?
   Which terms are divisible by 6?
   Which terms are divisible by 7?
   Which terms are divisible by 8?
   Which terms are divisible by 9?

   Complete this list:

   Every 3rd term is divisible by:
   Every 4th term is divisible by:
   Every 5th term is divisible by:
   Every 6th term is divisible by:

   Predict:

   Every 7th term is divisible by:
   Every 10th term is divisible by:

2. In the entire Fibonacci sequence there is only one perfect square (besides 1). What is it?

3. Instead of printing out F(I), print the ratio $\frac{F(I)}{F(I-1)}$.

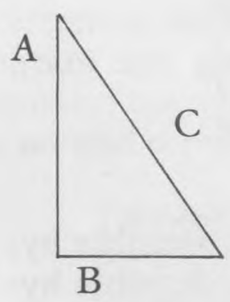
   Change the values of F(1) and F(2) to 2 and 7 and repeat.

   Change the values of F(1) and F(2) to any positive numbers and repeat.

   The number you keep finding is called the GOLDEN NUMBER. Compare it with (1+SQR(5))/2 or $\frac{(1+\sqrt{5})}{2}$

PYTHAGOREAN TRIPLETS

WARM-UP All right triangles satisfy the Pythagorean Theorem:



A and B are the legs

C is the hypoteneuse

$$A^2 + B^2 = C^2$$

If A, B, and C are integers, the triplet A,B,C is called a Pythagorean Triplet.

Problems

1. Write a program that begins:

```
10 PRINT "MAXIMUM LEG LENGTH"
20 INPUT L
30 FOR A = 1 TO L
40 FOR B = 1 TO A
50 D=A*A + B*B
60 C=SQR(D)
```

and tests whether or not C is an integer. *(Read the warning about the SQR function on page 43!)* If C is an integer, print out the values of A, B, and C. (It would be nice if you could only print those values of A,B, and C without a common factor.)

Let L be 50, and make a list of the values of A,B, and C that you find (list only those with no common factor).

The next questions refer to your list of triplets with no common factor. You may have to extend the list by running the program again with a larger L.

2. Is C always odd?

3. If C = A+1, what is true of B?

4. Is C always divisible by 5?

5. Will all odd numbers appear at least once as a leg? Find triplets with leg = 19, leg = 23. (Problem 3 may help here.)

6. For fixed C, are A and B uniquely determined?

7. Are there any integers which never appear in a triplet? If so, list them.

8. What numbers appear more than once as an A,B, or C? What are the prime factorizations of these numbers?

9. Are all the products of A*B*C divisible by 60?

LOCKER DOORS

WARM-UP These problems have to do with a long row of lockers numbered from 1 to 100. To refer to the lockers by number, we need to use an array.

L(1) refers to locker #1.
L(2) refers to locker #2.
L(3) refers to locker #3, etc.

L(1)=6 means there are 6 marks on locker #1.
L(2)=3 means there are 3 marks on locker #2.

To start with 100 lockers, all with clean doors, use these lines:

```
10 DIM L(100)       (no more than 100 lockers allowed)
20 FOR I = 1 TO 100
30 L(I) = 0
40 NEXT I
```

Setting: A school with 100 students and 100 lockers in a row.

The janitors open all the lockers and clean off the doors.

Action: One at a time, all 100 students come by:

Student #1 marks the door of every locker and closes each one.

Student #2 opens and marks the door of every second locker.

Student #3 marks the door of every third locker and changes the state of the door (opens it if it was closed, shuts it if it was open) and so on. Student #N changes the state and marks the door of every Nth locker.

To simulate the action, add these lines:

```
5Ø FOR N = 1 TO 1ØØ
6Ø FOR D = N TO 1ØØ STEP N
7Ø L(D) = L(D)+1            mark door of locker #D
8Ø NEXT D
9Ø NEXT N
```

Problems

1. Suppose, after all the students have gone by, a door has 5 marks on it. Is it open or closed? What if it has 4 marks?

2. After all 100 students have gone by, which lockers have closed doors? Write a program to answer this question.

3. Which doors have exactly two marks on them?

4. Which students touched the door of locker number 24?

Suppose there had been 1000 students and 1000 lockers. Try to answer these questions. (If your computer has enough memory you can check your answers.)

5. Which students touched locker #520?

6. How many divisors does 520 have?

7. There are 5 lockers which are touched exactly 14 times. One is locker #192. What are the others?

FACTORS AND MULTIPLES

WARM-UP To tell whether 7 is a factor of a 68257, divide 68257 by 7; check to see if the remainder is 0.

If the remainder is 0, then 68257/7 is an integer.

To check whether a number, X, is an integer, test whether X = INT(X).

Problems

1. The smallest divisor of any number is 1; the greatest is the number itself. If the computer tests all the numbers from 1 to N, it will find all the divisors, or factors, of N. Write a program that does this.

   Find all the divisors of 720 and 1350 and LIST them.

   720:

   1350:

2. The program can be speeded up by printing D, and N/D when a factor D of N is found. Change the PRINT statement in your program to print both numbers. Then decide how to change the upper limit in the

   ```
   FOR D = 1 TO N
   ```

   statement.

3. Now write a program to find the greatest common factor of two numbers, A and B. The program should start:

   ```
   10 PRINT "TELL ME TWO NUMBERS"
   20 INPUT A,B
   ```

   Check the factors of A to see whether they can also be divided into B. Find the GCF of 67831 and 72509.

4. The largest the least common multiple of two numbers can be is the product of the numbers. Write a program that begins like the one in #3, and which finds the LCM of A and B. Find the LCM of 2442 and 3848.

5. The television series "Special Hospital" had 236 episodes; the series "Turning World" had 136 episodes. Cable station WSOAP steadily broadcasts both series daily, starting each one over again as it finishes. Suppose on January 1, episode #1 of each series was shown. What is the next date on which episode #1 of both series will be shown?

APPLES AND BASEBALL CARDS

WARM-UP Suppose John and Pat had 177 apples between them, and that John had twice as many as Pat. How many did each one have?

A program to solve this problem could be:

```
10 FOR J=1 TO 177
20 FOR P=1 TO 177
30 IF J+P < > 177 THEN 90
40 IF J = 2*P THEN PRINT J,P
90 NEXT P
99 NEXT J
```

The computer is going to check a lot of numbers looking for the answer, but eventually, it will find it.

Problems

1. John, Pat, and Carol have 23 apples amongst themselves. John has 3 more apples than Carol. Pat has twice as many apples as Carol. How many apples does each have?

2. Paul, Jim, and Cynthia have fewer than 1000 baseball cards. Paul has twice as many as Jim, and 3 times as many as Cynthia. What are the possible numbers of cards belonging to each?

3. Mary is 6 years older than Susan. Right now Mary is twice as old as Susan. How old is each?

   A program to find Mary's and Susan's ages would start:

```
10 FOR M = 1 TO 80
20 FOR S = 1 TO 80
30 IF M=S+6 AND M=2*S THEN PRINT M,S
40 NEXT S
50 NEXT .....
```

4. Marty is 6 years older than Sam. Right now Marty is twice as old as Sam will be next year. How old is each right now?

5. Henry is 6 years older than Harriet. In two years Henry will be four times as old as Harriet was 4 years ago. How old are they now?

6. An athlete's training program calls for exactly 10 units of vitamin A, 9 units of vitamin D, and 19 units of vitamin E each day. There are three brands available with only vitamins A, D, and E: Xcellos, Yowees, and Zippyvites. Their composition and costs are:

Brand Name	Units of A	Units of D	Units of E	Cost per pill
X	1	3	4	3 cents
Y	2	3	5	6 cents
Z	1	0	1	2 cents

Find all possible combinations of pills that provide exactly the required amounts of vitamins. What is the daily cost of each combination?

7. A producer is arranging a 30 minute television show featuring a band, a comedian, and a singer. The combined time for the band and the comedian is twice the time allotted for the singer. The band gets 2 minutes more than the singer. The difference in time between the band and comedian is twice the difference between the singer and comedian. How many minutes does each act get?

FRACTIONS

WARM-UP When your computer adds fractions it needs a common denominator, but not necessarily the *least* common denominator. (Arithmetic is easy for a computer.)

To add $\frac{5}{6}$ and $\frac{8}{15}$

first write

$$\frac{5}{6}=\frac{5*15}{6*15}$$

$$\frac{8}{15}=\frac{8*6}{6*15}$$

so that $\frac{5}{6}+\frac{8}{15}=\frac{75+48}{6*15}$

To add $\frac{P}{Q}$ and $\frac{R}{S}$

first write

$$\frac{P}{Q}=\frac{P*S}{Q*S}$$

$$\frac{R}{S}=\frac{R*Q}{S*Q}$$

$$\frac{P}{Q}+\frac{R}{S}=\frac{P*S+R*Q}{Q*S}$$

Problems

1. Write a program to add and reduce to lowest terms, P/Q + R/S. Find the following sums, reduced to lowest terms.

$$\frac{1}{35}+\frac{3}{91}$$

$$\frac{3}{28}+\frac{6}{55}$$

2. If

$$\frac{1}{13}+\frac{1}{A}=\frac{1}{B}$$

then, since

$$\frac{1}{13}+\frac{1}{A}=\frac{A+13}{13*A}$$

it must be true that 13*A/(A+13) = INT(13*A/(A+13)).

Find integers A and B, A and B not the same, such that

$$\frac{1}{13}+\frac{1}{A}=\frac{1}{B}$$

3. Find integers A and B, A and B not the same, such that

$$\frac{1}{A}+\frac{1}{B}=\frac{1}{100}$$

4. Can you identify this fraction? It is both proper and reduced. Both numerator and denominator are less than 100. The difference between the denominator and numerator is a perfect square. If 3 is added to the numerator and 6 is subtracted from the denominator, the new fraction equals one half.

SQUARES (AND ROOTS)

WARM-UP

WARNING!

The SQR function is not always accurate. RUN this program which is supposed to find the square numbers between 1 and 200.

```
10 FOR I = 1 TO 200
20 IF SQR(I)=INT(SQR(I))
   THEN PRINT I
30 NEXT I
```

Which squares are missing?

To see what is happening, type

```
PRINT 10 - SQR(100)
```

The difference is small, but it's not 0.

To test whether a number is square, use this test in line 20:

```
20 IF ABS(SQR(I)-INT(SQR(I)))
   <.0001 THEN PRINT I
```

RUN the program again to make sure all the squares are printed.

Problems

1. How many ways can $5625 = 75^2$ be written as the sum of two squares? What are they?

2. There are two square numbers less than 10000 which can be written in 4 different ways as the sum of two squares. What are the numbers? (Heed the warning!)

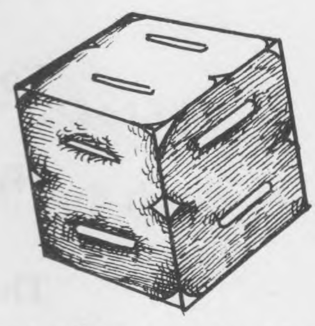
3. All the cubes can be written as the difference of two squares:

$1^3 = 1^2 - 0^2$

$2^3 = 3^2 - 1^2$

$3^3 = 6^2 - 3^2$

$4^3 = \_\_^2 - \_\_^2$



$5^3 = \_\_^2 - \_\_^2$

$6^3 = \_\_^2 - \_\_^2$

$7^3 = \_\_^2 - \_\_^2$

Find ways to write 4^3, 5^3, 6^3, 7^3 as the difference of two squares. A pattern will appear, but will seem to contain a mistake, because 6^3 can be written as the difference of squares in more than one way. Can you predict how to write 8^3 as the difference of two squares?

COIN TOSSING

WARM-UP To toss a coin, use the RND function. RND(1) is a number between 0 and 1; one way to toss a coin is to decide that if RND(1) > .5 then the toss was heads, otherwise it was tails.

Here's how to toss a coin 10 times.

```
10 FOR T = 1 TO 10
20 X = RND(1)
30 IF X>.5 THEN H = H+1
40 NEXT T
50 PRINT H, "HEADS"
```

Problems

1. Suppose you toss a coin 400 times. Every time it lands heads, you win $1. Every time it lands tails, you lose $1. Write a program that prints your score after each toss. (Use a semicolon(;) so you can see lots at once.)

   Record your final scores for 5 runs of the program.

2. Now write a program that plays the following game 400 times: 2 coins are tossed; if they match you win $1, otherwise you lose $1. RUN the program 6 times. Record your winnings or losses.

   If your average winnings, over thousands of games, are $0, then the game is *FAIR*. Does this look like a fair game?

3. Write a program to simulate the game of Gambler's Ruin: player A starts with $M and player B starts with $N. A coin is tossed. If it lands heads, A wins $1 from B, otherwise A gives $1 to B. The game ends when one of the players has all the money. Keep track of the length of the game, as well as who wins.

   You need only keep watch on player A: if he gets to $(M+N), he wins; if he gets to $0, he loses.

   Play twenty games with A starting with $6 and B with $4. Record how often A won and the length of each game.

CUBES & SQUARES

WARM-UP *Remember*, to your computer the square root of 100 is not 10.

To test whether a number N is a square, use:

```
IF ABS(INT(SQR(N))-SQR(N))
    <.0001 THEN ....
```

Sometimes it makes programming easier if you decide what to do when a number is *not* a perfect square.

Problems

1. Find the digit sum of every three-digit square. Which of the digit sums are not themselves squares?

2. Find all pairs of cubes whose sum is a square. Restrict your search to cubes less than 1000000 (100^3). Which pairs have no common factors? (Two odd numbers never total a square; two even cubes cannot total a square.)

3. Find all proper fractions of the form N/D which satisfy:

N+D	is one less than a square.
D–N	is one more than a square.
D*N	is one less than a square.
D	is less than 200.

   Is it always the case than D–N=2?

4. Find all proper fractions of the form N/D which satisfy:

N*D	is a multiple of 7.
N+D	is a square.
D–N	is no more than 3.
N/D	is in reduced form.

DEXIM PU ORSDW

WARM-UP To print the letters in a word, one at a time, try this:

```
1Ø PRINT "GIVE ME A WORD"
2Ø INPUT W$
3Ø FOR L = 1 TO LEN(W$)
4Ø PRINT MID$(W$,L,1)
5Ø NEXT L
```

To spell the word backwards, change line 30 to

```
3Ø FOR L = LEN(W$) TO
   1 STEP -1
```

If you use a semicolon, the letters will be printed on the same line.

One way to scramble a word is to give each letter a number. Then choose a number at random, print the letter associated with the number, and mark the number as already used. Continue to choose numbers; if a number chosen has been used, keep choosing until an unmarked number comes up.

This program will scramble words of 20 letters or fewer.

```
  5 DEF FNR(X) = INT(RND(1)*X+1)
 1Ø DIM LT(2Ø)
 2Ø PRINT "GIVE ME A WORD"
 3Ø INPUT W$
 4Ø L = LEN(W$)
 5Ø FOR J = 1 TO L
 6Ø K = FNR(L)
 7Ø IF LT(K)=1 THEN 6Ø
 8Ø PRINT MID$(W$,K,1);
 9Ø LT(K)=1
1ØØ NEXT J
```

There are L letters in a word.
K is a number from 1 to L; the Kth letter has been used, print the Kth letter, mark off the letter.

Try this program on short words like CART and MUSKRAT.

Then try it on longer words like STRENGTHENED and ALPHABETICALLY.

You can see the program slow down as it hunts for the last unused letter.

Problems

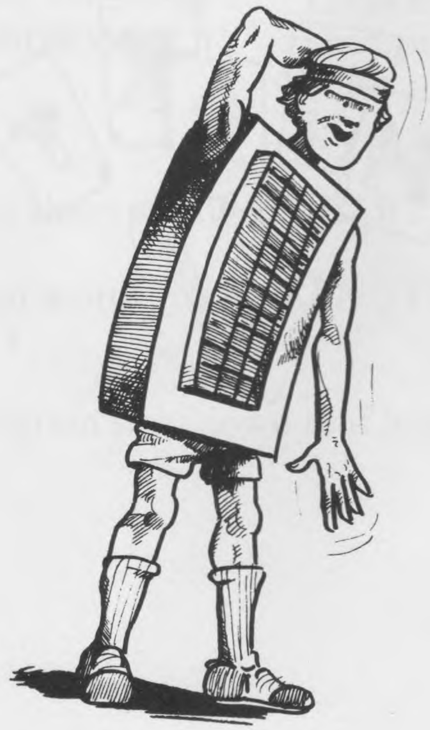
1. Write a guessing game. Give the player a mixed-up word to identify.

2. Can you write a program that prints out *all* the possible arrangements of a 4-letter word? (You won't use the random number generator here.) Try to figure out a system. If the letters are all different, how many arrangements are there?

TAKEN TWO AT A TIME

WARM-UP Here are two very different problems with the common requirement of taking two things at a time from a set of three.

If the set consists of A, B, and C, then the combinations of two things are: AB, AC, and BC.



Problems

1. Find three numbers such that their sum is a square, and the sum of any two of the numbers is also a square. (Hint: the largest number is less than 400.)

2. Once the lengths of the sides are known, a triangle is completely determined. Any three numbers determine a triangle, *provided that* the sum of the lengths of any two sides is greater than the length of the remaining side. Write a program that lets you input three numbers and tells you whether those numbers can form the sides of a triangle. Try your program on this set of numbers: 3, 4, 6, and on the set: 3, 4, 4.5.

COLLECTING SETS

WARM-UP Did you ever wonder how many boxes of cereal you'd have to eat to collect a complete set of six Space Wars figures? It's a good experiment to simulate, rather than carry out.

To keep track of how many of each figure you've collected, you need an array. F(1) tells how many of the first figure you have. F(2) tells how many of the second figure you have, etc.

Complete the program. RUN it 10 times, each for a set of six figures.

```
 10 DIM F(50)                          Maximum of 50 figures.
 20 PRINT "HOW MANY IN A SET"
 30 INPUT TF                           TF is total figures.
 40 DEF FNR(X)=INT(RND(1)*X)+1
 50 TC = 1                             TC is total cereal boxes.
 60 FG = FNR(TF)                       Open a box; get figure FG,
 70 F(FG)=F(FG)+1                      mark one more of figure FG.
 80 T=0
 90 FOR I = 1 TO TF
100 IF F(I)>0 THEN T=T+1
110 NEXT I                             Check how many different
120 IF T= TF THEN 200                  figures have been found
130 TC = TC + 1                        if the set is complete, END.
140 GOTO 60
200 PRINT TC
```

Problems

1. To make the computer collect 10 sets and do the averaging, you need to insert a FOR . . . NEXT loop:

```
55 FOR S = 1 TO 1Ø
22Ø NEXT S
```

You also need to clear the records of how many of each figure you accumulated the last time you collected a set.

```
15Ø FORI=1 TO TF
16Ø F(I) = Ø
17Ø NEXT I
```

Change line 120 to branch to 210 and keep track of the total number of cereal boxes:

```
21Ø TN = TN +TC
```

Print the average number of boxes, TN/10, in line 230.

```
23Ø PRINT ...........
```

2. Now try a set with 25 different characters. Keep track of your averages. The program will be more interesting to watch if it prints out the number of boxes needed for each set.

```
12Ø IF T=TF THEN PRINT T;" BOXES": GOTO 15Ø
```

Collect 10 sets of 25 figures and compute the average number of boxes required for one set.

SUCCESS AT LAST

WARM-UP To make the computer think of a number from 1 to 6, use the statement:

```
X = INT(RND(1)*6+1
```

Here's how to roll a die 10 times:

```
1Ø FOR T = 1 TO 1Ø
2Ø X = INT(RND(1)*6+1)
3Ø PRINT X
4Ø NEXT T
```

Problems

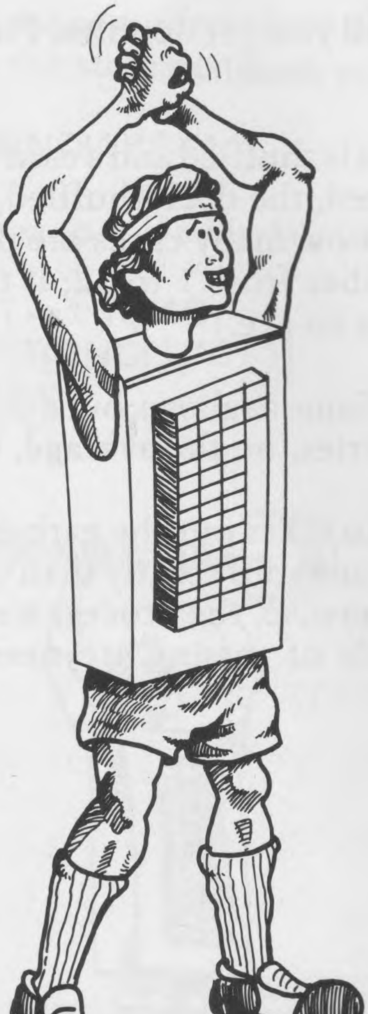
1. Keep rolling a die until you get a 3. Keep track of how many rolls it took. Do this 10 times (if you can make the computer keep track, do it 100 times) and calculate the average number of rolls until you get a 3.

2. Roll two dice until you get doubles. Find the average number of rolls needed to get doubles.

3. A deck of 52 cards is shuffled and a card is drawn. If it is not an ace, the card is replaced, the deck shuffled, and another card drawn. On the average, how many cards are drawn to get the first ace? (Generate a number from 1 to 52; if the number is 1,2,3, or 4, then the card was an ace.)

   CHALLENGE   Same as above, but a drawn card is NOT replaced. Now how many tries, on the average, to get an ace?

4. To determine who takes out the garbage, five people each toss a coin. If one coin lands differently than the other four, that person does the job; otherwise, the process is repeated. On the average, how many rounds of tossing are needed for a decision to be made?

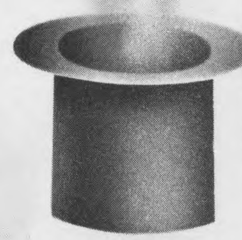


WRAP-UPS

or

Second Helpings

For those who want some more — continuations, explanations, variations, extensions, and digressions based on the Problem Sets.



WRAP-UP
SHOPPING PROBLEMS

Equations of the form

$$AX + BY = C$$

where A,B,C,X, and Y are integers, are called *Diophantine equations.* Diophantus was a Greek living in Alexandria in about 250 A.D. Virtually all that is known about his life comes from a Greek collection of problems:

His boyhood lasted 1/6 of his life,

His beard grew after 1/12 more,

He married after 1/7 more,

5 years later his son was born,

The son died when he was exactly half his father's age,

His father died 8 years later.

How long did Diophantus live?

The first three clues will answer the question, as all the numbers referred to are integers.

Look at the Diophantine equation

$$32P + 88N = C$$

The greatest common divisor of 32 and 88 is 8. Factor 8 from the left-hand side:

$$8(4P + 11N) = C$$

C must be a multiple of 8. In general, if

$$AX + BY = C$$

has a solution, then C is a multiple of the GCD of A and B.

If negative values of X and Y are allowed, then the GCD itself is a possible C:

$$32 * 3 + 88 * (-1) = 8$$

Since this equation can be multiplied by 2, 3, 4, and so on, it follows that *all multiples of the GCD are possible values of C.*

If negative X and Y are not allowed (and in the shopping problems they weren't), then every possible C is a multiple of the CGD, but not all multiples are possible C's.

Final problem: which multiples of 8 are possible solutions to

$$32P + 88N = C \quad ?$$

WRAP-UP REMAINDERS

Numbers which leave the same remainder when divided by 7 are said to be *congruent modulo 7*. In the pirates problem, if G is the number of gold pieces, then:

G is contruent to 1 modulo 12;

G is congruent to 6 modulo 11;

G is congruent to 9 modulo 10.

This is called a system of congruences.

The (advanced) mathematical method for solving systems of congruences is called the *Chinese Remainder Theorem*. You'll see it in college if you take a course in Number Theory or Abstract Algebra.

A very different problem is that of the pirates, the monkey, the coconuts, and the desert island:

Five pirates spend the day gathering coconuts. They make a large pile, then go to sleep.

The first pirate awakens, divides the coconuts into 5 equal piles with 1 left over. He hides his share, piles up the rest of the nuts, gives the extra coconut to the monkey who eats it, and throws away the shell. The pirate then goes back to sleep.

The second man awakens, takes one fifth of the remaining coconuts from the pile; there is 1 left over which he too gives to the monkey. He goes back to sleep.

The third, fourth, and fifth pirate do the same in turn (the pile of coconuts getting smaller each time). Each time, there is 1 left over which is tossed to the monkey.

Find the minimum number of coconuts in the original pile.

WRAP-UP
ADDING AND AVERAGING

A computer can add up only a finite number of things. However, there are some sums, like

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

which, although they have an infinite number of terms, actually have a finite value. You can get *very* close to the sum by adding up a lot of terms and ignoring the rest. (How many terms constitute "a lot" depends on the particular sum you're using.)

Each term in the sum is exactly (½)*previous term. To add the first 10 terms and see the sum as it's being computed, use this program:

```
10 S = 0
20 T = 1/2
30 FOR N = 1 TO 10
40 S = S + T
50 T = T*(1/2)
60 ?N,S
70 NEXT N
```

<u>Problem</u>: Add the first 30 terms of each of these:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots$$

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots$$

$$\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\frac{1}{625}+\cdots$$

Try to write each answer you got as a fraction.

Predict the sum of:

$$\frac{1}{6}+\frac{1}{36}+\frac{1}{216}+\frac{1}{1296}+\cdots$$

and check your prediction.

WRAP-UP
DIMINISHING RETURNS

If your bank pays 8% interest per year, compounded monthly, it splits 8% into 12 parts and each month gives you 8/12 % interest on your balance.

To print out the monthly balance, assuming the starting balance was $1000, try:

```
10 BL = 1000
20 FOR M = 1 TO 12
30 I = BL*(.08/12)
40 BL = BL + I
50 PRINT M, BL
60 NEXT M
```

Did the balance reach $1100 in one year?

If the form of BL bothers you, you can get rid of the decimals with

```
45 BL = INT(BL*100)/100
```

This won't round the cents, but otherwise it will work. (Nothing reasonable will add a final 0 after a decimal point in BASIC.)

Try these problems. They all assume interest is 8%, compounded monthly.

How many months does it take until the $1000 doubles?

Add line 35 so that after the interest is computed, you can increase the balance by making a deposit. If you start with $1000 and add $50 each month, how much will you have after 48 months?

How much must you deposit each month to have at least $6000 at the end of 4 years?

WRAP-UP
DIGITS

In the days before calculators, the digit sum was used in *Casting Out Nines.* The process tells whether a number is divisible by 9. It can also be used to make a rough check of a lengthy addition problem.

Casting Out Nines works like this:

Take a number, say 45678834

Find the digit sum;
4+5+6+7+8+8+3+4= 45

If the digit sum is more than 10, find its digit sum. 4+5=9

Keep going until the digit sum is less than 10.

If the last sum is 0 or 9, then the original number was divisible by 9.

The computer can do casting out nines on enormous numbers — much longer than the 10 digits it uses when it does arithmetic. Enter the number as a string; then use MID$ and VAL to get the digit and their values.

```
 10 PRINT "WHAT'S THE NUMBER"
 20 INPUT N$
 30 S = 0
 40 FOR D = 1 TO LEN(N$)
 50 S = S + VAL(MID$(N$,D,1))
 60 NEXT D
 70 PRINT S
 80 IF S<10 THEN END
 90 N$ = STR$(S)
100 GOTO 30
```

Line 50: The value of the digit is added.

Lines 90–100: Turn the digit sum into a string; find the next digit sum.

Find out whether these numbers are divisible by 9:

3794682371583147   854123678455631

If the digit sum of a number is 0, 3, 6, or 9 (if it's divisible by 3), then the number was divisible by 3. Test these for divisibility by 3:

54897632253   12345678987654321

WRAP-UP
MAKING CHANGE

In the land of Binaree, there are coins worth 1 cent, 2 cents, 4 cents, 8 cents, 16 cents, 32 cents, and 64 cents. Coins are so rare that a citizen is subject to arrest if he has more than one of each coin. Consequently, each citizen carries paper money and one coin of each denomination; when they spend coins they can replace them at the treasury. How would the citizen pay for each of the following items?

A pen costing 39 cents?

A notebook costing 88 cents?

A lightbulb costing 97 cents?

In the land of MSU there are only two kinds of paper money, sickels which are worth 7 dollars, and nimes which are worth 9 dollars. The people of MSU have become very clever at making change. Pat wanted to buy a radio which cost $20. This is how the transaction proceeded:

Pat gave the shopkeeper 3 nimes ($27).

The shopkeeper gave Pat the radio ($20) and one sickel ($7).

What does Pat give the shopkeeper, and what does Pat get back in change for each of the following purchases:

A $29 watch?

A $40 tennis racket?

A $300 moped?

(These are tough problems to put on a computer. If you can make the computer solve them, you're pretty sharp.)

WRAP-UP
FIBONACCI SEQUENCE

The real name of Fibonacci was Leonardo of Pisa, son of Bonacci. In 1202 he wrote about a population of rabbits. His rabbits did not reproduce until they were 2 months old, then they produced a new pair of rabbits every month. He counted pairs of rabbits, rather than individual rabbits.

Let F(1)	be	the	total	pairs	at	the	beginning	of	month 1.
F(2)	"	"	"	"	"	"	"	"	2.
F(3)	"	"	"	"	"	"	"	"	3.
F(4)	"	"	"	"	"	"	"	"	4.
F(5)	"	"	"	"	"	"	"	"	5.

and suppose we start with 1 pair of baby rabbits.

F(1) = 1	
F(2) = 1	since the pair isn't mature (2 months old).
F(3) = 1 + 1 = 2	the old pair and 1 new pair.
F(4) = 2 + 1 = 3	only the old pair is mature.
F(5) = 3 + 2 = 5	the 3 were already here. The 2 pairs who were alive 2 months ago are now mature.

In general:

$$F(M) = F(M-1) + F(M-2)$$

Your list of Fibonacci numbers can be regarded as a list of the rabbit population at the beginning of each month. These rabbits never die.

Change the equation so that every pair that was alive 8 months ago dies right after producing its last pair.

$$F(M) = F(M-1) + F(M-2) - F(M-8)$$

Make a list of the monthly totals for the rabbits now.

(Be careful how you start the program!)

WRAP-UP
PYTHAGOREAN TRIPLETS

If A, B, and C satisfy $C^2 = A^2 + B^2$, then A, B, and C are the sides of a right triangle; C is opposite the right angle.

If A, B, and C satisfy $C^2 = A^2 + B^2 - 1$, then A, B, and C are the sides of a 60 degree triangle; C is opposite the 60-degree angle.

If A, B, and C satisfy $C^2 = A^2 + B^2 + 1$, then A, B, and C are the sides of a 120 degree triangle; C is opposite the 120 degree angle.

Find some 60 and 120 degree triangles with integer sides.

WRAP-UP
LOCKER DOORS

Change the locker problem so that the number of marks made by each student is the same as that particular student's number.

Student #1 makes 1 mark on each door she touches.
Student #2 makes 2 marks on each door he touches.
Student #3 makes 3 marks on each door she touches.
Student #4 makes 4 marks on each door he touches.

Use at least 1000 lockers.

The number of marks on some lockers will be exactly twice the locker number. Which lockers are these?

The number of marks on some lockers will be exactly three times the locker number. Which lockers are these?

A number which is exactly equal to the sum of its proper divisors (counting 1, but not including the number itself) is called a *perfect number*. What perfect numbers have you found?

WRAP-UP
FACTORS AND MULTIPLES

Try to write a program that finds the prime factors of a number. Remember that after 2, all the primes are odd.

An outline of the program could be:

Enter N (the number)

*Test whether N is divisible by 2:
If yes, then print 2. Replace N with N/2.
Test for divisibility by 2 again. [GOTO *]
If no, continue

Set D = 3 (D is a possible divisor of N)

**Test whether N is divisible by D:
If yes, then print D. Replace N by N/D.
Test for divisibility by D again. [GOTO **]
If no, continue

D = D + 2 (look at next odd number).

Test D. If D < test number, GOTO **

One test for D is to see whether D > SQR(N).

Find the prime factorizations of these numbers:

1001
100001
488234

Write a program so that it only prints out the primes from 2 to 1000. What's the largest prime less than 1000?

* and ** represent line numbers.

WRAP-UP
APPLES AND BASEBALL CARDS

The problems in this set can each be represented by a system of equations. In Problem 1, if John has J apples, Pat has P apples, and Carol has C apples, the equations are:

$$\begin{aligned} J + P + C &= 23 \\ J &= C + 3 \\ P &= 2*C \end{aligned}$$

You used the equations to test possible solutions. In an algebra class, the equations would be written with all the unknowns on the left-hand side:

$$\begin{aligned} J + P + C &= 23 \\ J \quad\;\; - C &= 3 \\ P - 2C &= 0 \end{aligned}$$

Some sets of equations have exactly one solution, some have many. Problems 1, 3, 4, and 5 each had a unique solution.

Try this old Chinese problem, called the *"Problem of 100 Fowl."*

> If a rooster is worth 5 coins,
> if a hen is worth 3 coins,
> if 3 chicks are worth 1 coin,

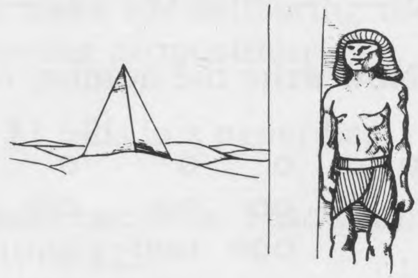
how many of each kind must you have so that:

> There are 100 fowl in all,
> there are some of each kind,
> the fowl are worth 100 coins?

WRAP-UP FRACTIONS

For some reason, the Egyptians always wrote fractions as the sum of distinct unit fractions. For example:

$$\frac{3}{7}=\frac{1}{4}+\frac{1}{7}+\frac{1}{28}$$



They also could have written:

$$\frac{3}{7}=\frac{1}{4}+\frac{1}{12}+\frac{1}{11}+\frac{1}{231}$$

Find two Egyptian fractions of length 3, representing:

$$\frac{8}{13}$$

You can use pencil, paper, and the fact that:

$$\frac{1}{X}=\frac{1}{X+1}+\frac{1}{X(X+1)}$$

to do something like this:

$$\frac{3}{7}=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{1}{7}+\frac{1}{8}+\frac{1}{56}+\frac{1}{8}+\frac{1}{56}$$
$$=\frac{1}{7}+\frac{1}{8}+\frac{1}{56}+\frac{1}{9}+\frac{1}{72}+\frac{1}{57}+\frac{1}{3192}$$

This can take a long time, and will probably lead to very long Egyptian fractions.

WRAP-UP
SQUARES AND ROOTS

Write the difference between each pair of numbers in the sequence:

1 __ 3 __ 6 __ 10 __ 15 __ 21 __ 28 __ 36 __ 45 __ 55 __ 66 __ 78

Now write the number of balls in each triangle below:

```
o o   o    o     o      o       o        o
  oo  oo   oo    oo     oo      oo       oo
      ooo  ooo   ooo    ooo     ooo      ooo
           oooo  oooo   oooo    oooo     oooo
                 ooooo  ooooo   ooooo    ooooo
                        oooooo  oooooo   oooooo
                                ooooooo  ooooooo
                                         oooooooo
```

The numbers in the sequence 1, 3, 6, 10, . . . are called *triangular numbers.* They pop up a lot in mathematics.

There is a formula for the N^{th} triangular number:

$$1 + 2 + 3 + 4 + \ldots + N = N(N + 1)/2$$

WRAP-UPS
COIN TOSSING

Here's another situation that might be fair or unfair. Henry's customers are supposed to pay him $5 each week for delivering the paper. One customer makes him the following proposition:

> Each week I'll put one $10 and five $1 bills in a paper bag.
>
> You reach in, without looking, and take two bills. That's your pay for the week. You could get $11, or $2.
>
> We'll stick with this method of payment for 52 weeks — that's one year.

To simulate this you'll need two RND values. The first will generate a 1, 2, 3, 4, 5, or 6 (corresponding to the six bills in the bag). If a 1 turns up, assume Henry was lucky and pulled the $10. In that case you know he gets $11 for the week.

If he didn't get the $10 on the first draw, he might get it on the second. Now there are only 5 bills left, so you have to generate a 1, 2, 3, 4, or 5. Again, let the 1 represent the $10.

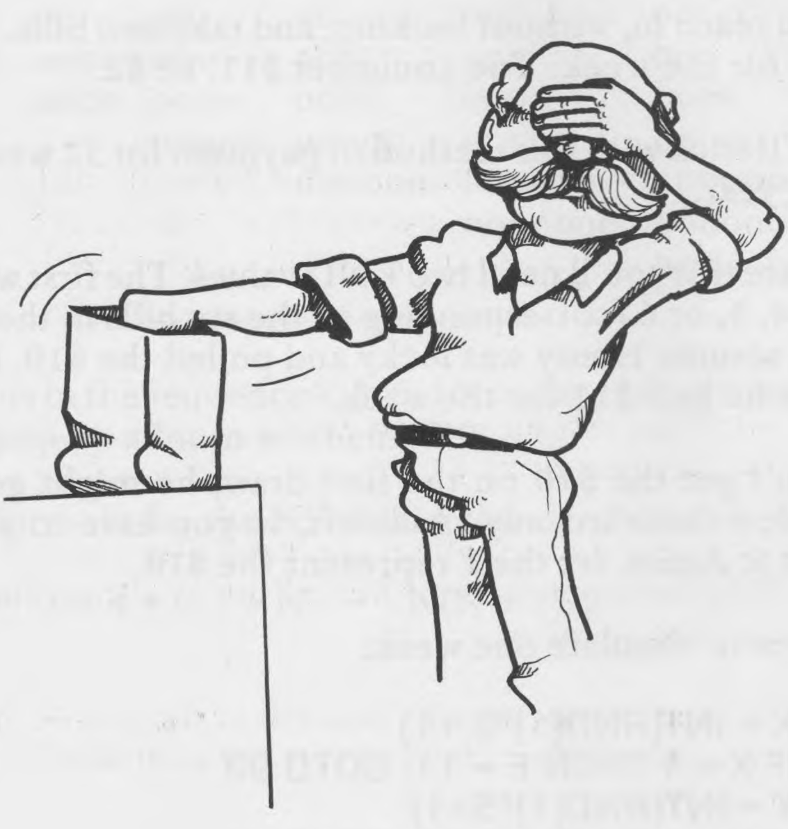
Here's how to simulate one week:

```
10 X = INT(RND(1)*6 +1)
20 IF X = 1 THEN E = 11: GOTO 90
30 Y = INT(RND(1)*5+1)
40 IF Y = 1 THEN E = 11: GOTO 90
50 E = 2
90 PRINT E
```

RUN the program for 52 weeks. Compare Henry's total earnings with 52*5 = 260 which is what the usual payments would total.

Does this seem to be fair?

Another customer, hearing of the game, suggests to Henry that she'll put one $5 and three $1 bills in a bag. The same procedures would be followed. Simulate this arrangement and decide whether or not it's fair.



WRAP-UPS
CUBES AND SQUARES

Which numbers *cannot* be written as the difference of two squares? The program:

```
10 FOR A = 1 TO 17
20 FOR B = 0 TO 1
30 PRINT A*A–B*B;
40 NEXT B
50 NEXT A
```

will fill your screen with numbers which are the difference of two squares. What numbers aren't there? (You can be sure about those less than 30; they would have appeared in your list.)

All these numbers can be written as ________________
(Look at the factorizations.)

Now look at the numbers which can be written as the sum of two squares? What numbers cannot be written as either the sum or the difference of two squares?

The numbers that cannot be written as either the sum or the difference of two squares are the numbers that leave a remainder of ________ when divided by ________.

WRAP-UPS
DEXIM PU ORDSW

It took a long time to find the last unused letters in ALPHABETICALLY. There's a way to make the computer select its next letter *only* from the unused letters. To do this, you have to keep switching the unused letters to the top of the list.

To switch the values of two variables A and B, you need to use a temporary storage variable, T.

T = A	Hold the value of A
A = B	A now has the value of B
B = T	what was originally in A is now in B.

This program will scramble letters very fast.

```
 5 DIM L$(2Ø)
1Ø A$ = "ALPHABETICALLY"
2Ø L = LEN(A$)
```

3Ø FOR I = 1 TO L 35 L$(I) = MID$(A$,I,1) 38 NEXT I	Make the list whose entries are the letters of A$.
5Ø FOR N = L TO 1 STEP −1 6Ø X = INT(RND(1)*N+1) 7Ø PRINT L$(X)	First check the entire list, then all but the last entry, then all but the last 2 entries, and so on.

```
80 T$ = L$(X)
82 L$(X) = L$(N)
84 L$(N) = T$
```

Move the last unused letter up the list, move L$(X) to the end of the last group of N letters.

```
90 NEXT N
```

When the program is done, have it print L$(I); for I=1 to L. The array still has all the letters of A$, but they've been rearranged.

To get several rearrangements of A$, put a FOR . . . NEXT loop in lines 45 and 95.

```
45 FOR S = 1 TO 5
90 NEXT S
```

Change A$ to GALE and let S go to 24. How many arrangements were repeated?

Change A$ to LARGE. There are 120 possible arrangements of 5 different letters. If you generate 50, do you get a repeat?

WRAP-UPS
TAKEN TWO AT A TIME

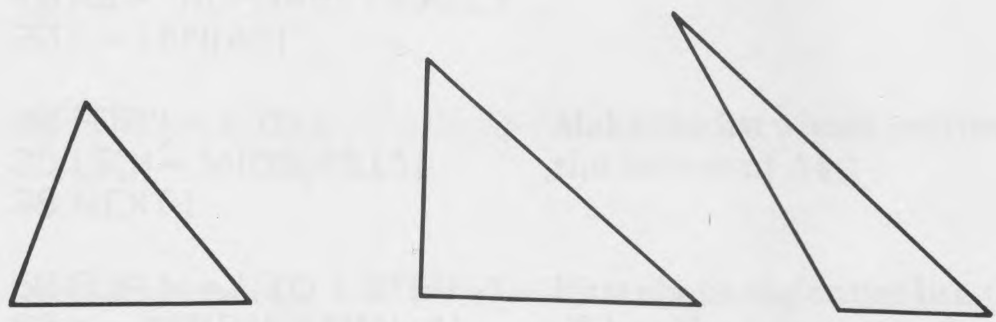
The first problem was posed by Diophantus (see the wrap-up for Shopping Problems). Here's a problem about triangles that Diophantus considered:

> Find a right triangle such that the hypoteneuse minus each of the sides gives a cube.

The second problem can be extended to classifying triangles. Suppose you have numbers A, B, and C which satisfy:

$$A + B > C$$
$$B + C > A$$
$$C + A > B$$

(that condition is known as the "triangle inequality"). Look at these three triangles

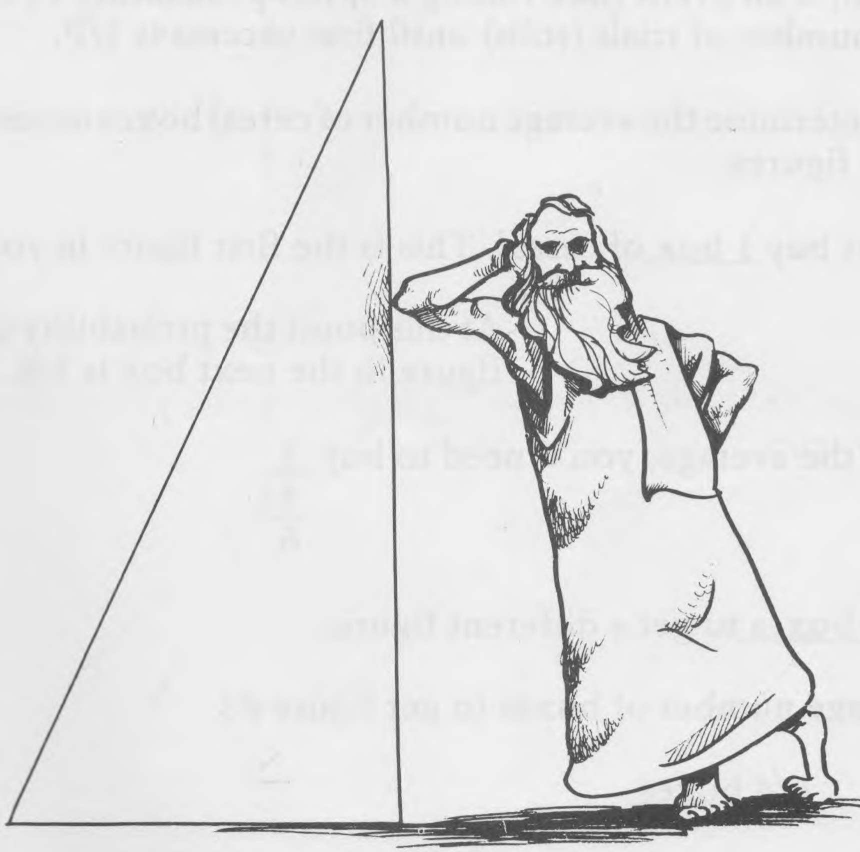


If the largest angle is less than a right angle, the triangle is *scalene*. If the largest angle is a right angle, then the triangle is a *right triangle*. If the largest angle is greater than a right angle, then the triangle is *obtuse*.

If C is the *longest* side, then if:

$C^2 < A^2 + B^2$ the triangle is scalene
$C^2 > A^2 + B^2$ the triangle is obtuse
and if $C^2 = A^2 + B^2$ the triangle is a right triangle.

Change your program to classify any triangle it finds.



WRAP UP
COLLECTING SETS

In Success at Last!, you should find that the average number of rolls of a die to get a 3 is 6. Every time you roll a die, the chances of getting a 3 are 1 in 6. The probability of getting a 3 is 1/6.

In general, if an event (like rolling a 3) has probability P, then the average number of trials (rolls) until first success is 1/P.

We can determine the average number of cereal boxes necessary for a set of 6 figures:

First buy 1 box of cereal. This is the first figure in your set.

At this point the probability of a new figure in the next box is 5/6.

On the average, you'll need to buy $\frac{1}{\frac{5}{6}}$

or 6/5 boxes to get a different figure.

The average number of boxes to get figure #3

is 6/4 boxes

You'll need
6/3 boxes for figure #4,
6/2 boxes for figure #5, and
6/1 boxes for figure #6.

If you add all the average numbers of boxes, you've bought:

$$\frac{6}{6} + \frac{6}{5} + \frac{6}{4} + \frac{6}{3} + \frac{6}{2} + \frac{6}{1} \quad = 6^{*}\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

boxes.

Write a program to evaluate the right hand side of the equation. Make the changes necessary to get the average number of boxes bought when there are 25 figures in the set.

WRAP-UP
SUCCESS AT LAST

The chances of a fair coin landing heads are 1 out of 2. Another way to say this is:

The probability of heads is 1/2.

Similarly,

The probability of rolling a 3 is 1/6 and
The probability of rolling doubles is 6/36 = 1/6.

The <u>Average number of trials</u> for something to happen is the reciprocal of its probability (1 divided by the probability).

Look at three tennis players, Bill, Will, and Phil, who are trying to decide who sits out the first set. They decide that each will toss a coin. If one coin lands differently from the others, that person will sit out the first set. Let's look at every possible way the coins can land.

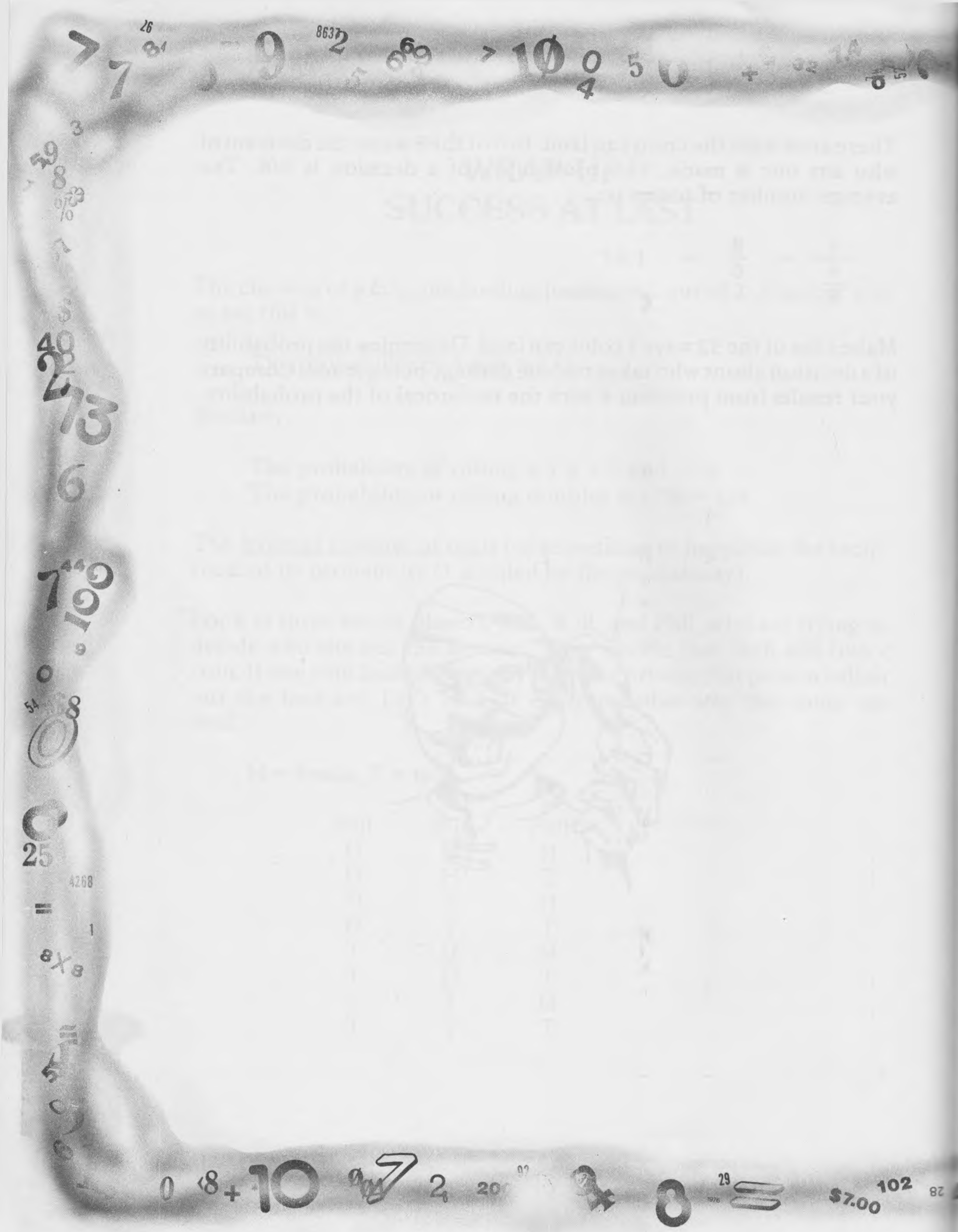
H = heads, T = tails

Bill	Will	Phil
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

There are 8 ways the coins can land. In 6 of the 8 ways, the decision of who sits out is made. The probability of a decision is 6/8. The average number of tosses is:

$$\frac{1}{\frac{6}{8}} = \frac{8}{6} = 1.67$$

Make a list of the 32 ways 5 coins can land. Determine the probability of a decision about who takes out the garbage being made. Compare your results from problem 4 with the reciprocal of the probability.



COUPON

COMPUMATH MAGIC

Diskette

Please send me the Compumath Magic diskette for my:

☐ Apple ☐ Atari ☐ Commodore-64

Enclosed is $15.95 per diskette
$2.00 shipping/handling charge
California residents please include 6.5% sales tax ($1.04)

COUPON

COUPON

Name ______________________________

Address ______________________________

City ____________ State ____________ Zip ____________

☐ Check ☐ Mastercard ☐ VISA

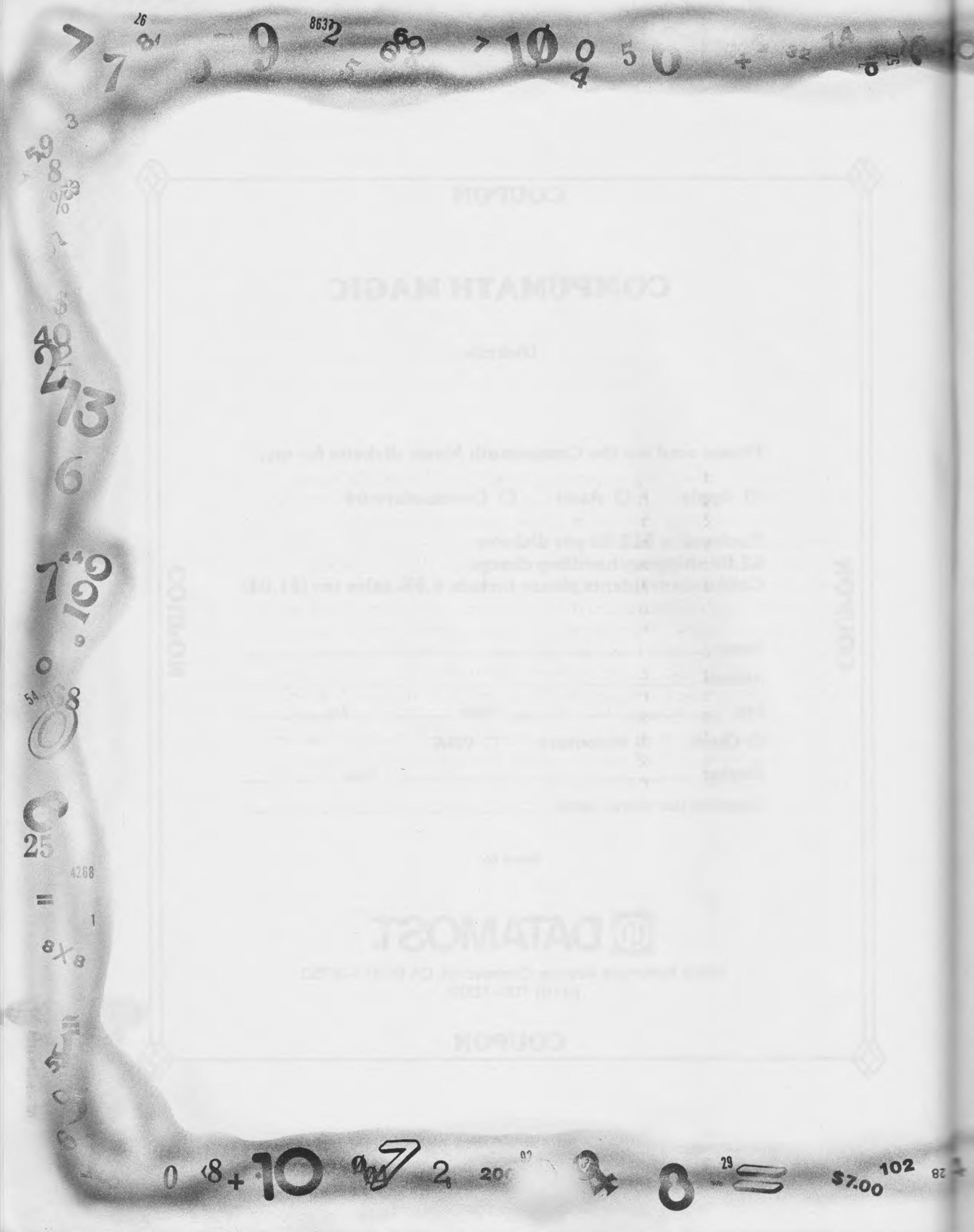
Number ______________________ Exp. ____________

Signature (for charge cards) ______________________

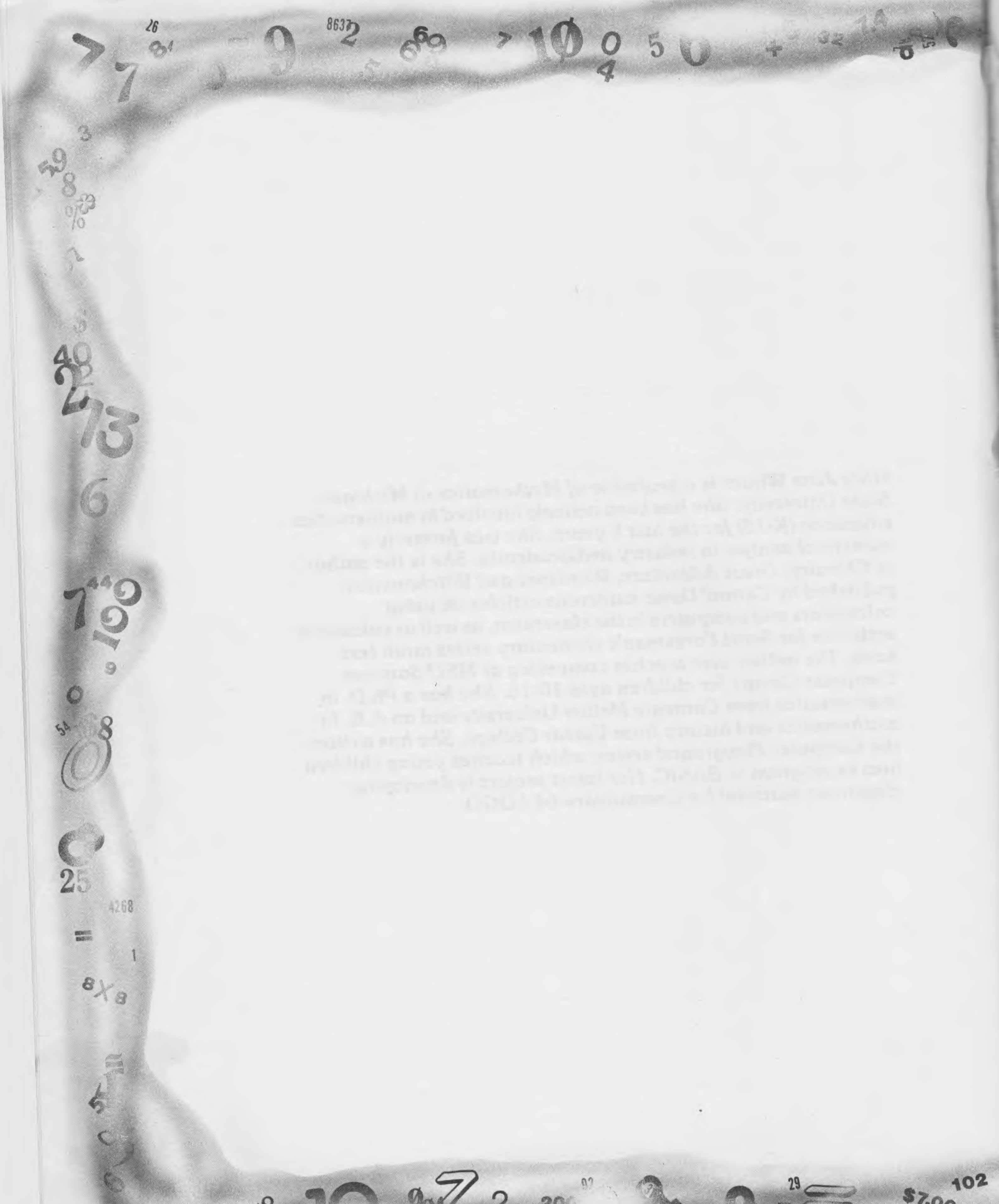
Send to:

DATAMOST™ INC
8943 Fullbright Avenue, Chatsworth, CA 91311-2750
(818) 709-1202

COUPON



*Mary Jean Winter is a professor of Mathematics at Michigan State University. She has been actively involved in mathematics education (K-12) for the last 8 years. She was formerly a numerical analyst in industry and academia. She is the author of Chivalry, Great Adventure, Wordspot and Witchnumber, published by Comm*Data; numerous articles on using calculators and computers in the classroom, as well as calculator activities for Scott Foresman's elementary series math text book. The author also teaches computing at MSU Summer Computer Camps for children ages 10-16. She has a Ph.D. in mathematics from Carnegie Mellon University and an A.B. in mathematics and history from Vassar College. She has written the Computer Playground series, which teaches young children how to program in BASIC. Her latest project is developing classroom material for Commodore-64 LOGO.*



Compumath Magic

Designed to run on ANY microcomputer, COMPUMATH MAGIC was written for "intermediate" programmers who want to learn how to use BASIC in varying applications. All that's required is a working knowledge of 6th grade math and the desire to learn how a computer can be used for problem-solving.

Books of computer problems are often hodgepodge collections. COMPUMATH MAGIC is arranged so that the problems in each set are related, but progress in complexity. Each set begins with a warm-up exercise, a problem set, and a corresponding wrap-up section. Successive problems require moderate, then greater modifications of the original problem. Earlier problems demonstrate HOW to carry out certain procedures, and later problems require decisions about WHAT procedures are needed. By the time you're finished, you'll have a working knowledge of the power and applicability of many mathematical techniques.

And, to help make things even clearer for those needing help getting started, a disk with at least one program for each problem set is available.

There's a difference between knowing BASIC and knowing how to use it to solve problems. COMPUTMATH MAGIC teaches BASIC problem-solving to everyone from kids to adults!

ISBN 0-88190-217-9

0 48831 00042

DATAMOST™ INC

8943 Fullbright Avenue, Chatsworth, CA 91311-2750
(818) 709-1202